NOTES

- NOTES WITH MIND MAPS -

MATHEMATICS

(RATIONAL NUMBERS)

7TH

Rational Numbers

1. The numbers of the form $\frac{x}{y}$, where x and y are natural numbers, are known asfractions.

2. A number of the form $\frac{p}{q}$[q × 0], where p and q are integers, is called a rational number.

3. Every integer is a rational number and every fraction is a rational number.

4. A rational number $\frac{p}{q}$ is positive if p and q are either both positive or both negative.

5. A rational number $\frac{p}{q}$ is negative if one of p and q is positive and the other one is negative.

6. Every positive rational number is greater than 0.

7. Every negative rational number is less than 0.

8. If $\frac{p}{q}$ is a rational number and 'm' is a non-zero integer, then $\frac{p}{q} \times \frac{p \times m}{q \times m}$. Here, $\frac{p}{q}$ and $\frac{p \times m}{q \times m}$ are known as equivalent rational numbers.

9. If $\frac{p}{q}$ is a rational number and 'm' is a common divisor of p and q, then $\frac{p}{q} \times \frac{p \times m}{q \times m}$. Here, $\frac{p}{q}$ and $\frac{p \times m}{q \times m}$ are known as equivalent rational numbers.

10. Two rational numbers are equivalent only when the product of numerator of the first and the denominator of the second is equal to the product of the denominator of the first and the numerator of the second.

11. A rational number $\frac{p}{q}$ is said to be in standard form if q is positive and the integers p and q have no common divisors other than 1.

12. If there are two rational numbers with a common denominator then the one with the largernumerator is greater than the other.

13. Rational numbers with different denominators can be compared by first making their denominators same and then comparing their numerators.

14. There are infinite rational numbers between two rational numbers.

15. Two rational numbers with the same denominator can be added by adding their numerators, keeping the denominator same.

$$\frac{p}{q} \times \frac{r}{q} \times \frac{[p \times r]}{q}$$

16. Two rational numbers with different denominators are added by first taking the LCM of the two denominators and converting both the rational numbers to their equivalent forms having the LCM as the denominator.

17. While subtracting two rational numbers, we add the additive inverse of the rational number to be subtracted to the other rational number.

$\frac{p}{q} \times \frac{r}{s} \times \frac{p}{q} \times$ (additive inverse of $\frac{r}{s}$)

18. To multiply two rational numbers, we multiply their numerators and denominators separately, and write the product as $\frac{\text{product of numerators}}{\text{product of denominators}}$

19. Reciprocal of $\frac{p}{q}$ is $\frac{q}{p}$.

20. To divide one rational number by the other non-zero rational number, we multiply the first rational number by the reciprocal of the other.

Rational Numbers

A rational number is defined as a number that can be expressed in the form

$$\frac{p}{q}$$

, where p and q are integers and q≠0.

In our daily lives, we use some quantities which are not whole numbers but can be expressed in the form of

$$\frac{p}{q}$$

Hence, we need rational numbers.

Equivalent Rational Numbers

By multiplying or dividing the numerator and denominator of a rational number by a same non zero integer, we obtain another rational number equivalent to the given rational number. These are called equivalent fractions.

$\left[\frac{1}{3} = \frac{1}{3} \times \frac{2}{2} = \frac{2}{6}\right]$

$\therefore$

$\frac{2}{6}$ and $\frac{1}{3}$ are equivalent fractions.

$\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$

$\therefore$

$\frac{15}{25}$ and $\frac{3}{5}$ are equivalent fractions.

Rational Numbers in Standard Form

A rational number is said to be in the standard form if its denominator is a positive integer and the numerator and denominator have no common factor other than 1.

Example: Reduce

$\frac{-4}{16}$

Here, the H.C.F. of 4 and 16 is 4.

$\Rightarrow \frac{-4}{16} = \frac{\frac{-4}{4}}{\frac{16}{4}} = \frac{-4}{16}$

$\frac{-1}{4}$

is the standard form of

$\frac{-4}{16}$

LCM

The least common multiple (LCM) of two numbers is the smallest number (≠0) that is a multiple of both.

Example: LCM of 3 and 4 can be calculated as shown below:

Multiples of 3: 0, 3, 6, 9, 12, 15

Multiples of 4: 0, 4, 8, 12, 16

LCM of 3 and 4 is 12.

Rational Numbers between Two Rational Numbers

There are unlimited number (infinite number) of rational numbers between any two rational numbers.

Example: List some of the rational numbers between −35 and −13.

Solution: L.C.M. of 5 and 3 is 15.

⇒ The given equations can be written as

$$\frac{-9}{15}$$

and

$$\frac{-5}{15}$$

⇒ −615, −715, −815 are the rational numbers between −35 and −13.

Note: These are only few of the rational numbers between −35 and −13. There are infinte number of rational numbers between them. Following the same procedure, many more rational numbers can be inserted between them.

Properties of Rational Numbers

- **Closure Property**
  Sum, difference and product of two rationals is again a rational number. So, Rational numbers are closed under addition, subtraction, multiplication but **NOT** under division.

- **Commutativity Property**
  For any two rational numbers a and b $a * b = b * a$.
- Rational numbers are commutative under addition and multiplication but **NOT** under subtraction and division.
  Example: $\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$ and $\frac{3}{7} + \frac{1}{7} = \frac{4}{7}$
  $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18} = \frac{5}{9}$ and $\frac{5}{6} \times \frac{2}{3} = \frac{5}{9}$
  $\frac{1}{2} - \frac{3}{4} = -\frac{1}{4}$ but $\frac{3}{4} - \frac{1}{2} = \frac{1}{4}$
  $\frac{3}{7} \div \frac{5}{2} = \frac{6}{35}$ but $\frac{5}{2} \div \frac{3}{7} = \frac{35}{6}$

- **Associative Property**
  For any three rational numbers a, b and c, $(a * b) * c = a * (b * c)$.
- Addition and multiplication are associative for rational numbers, but subtraction and division are **NOT** associative for rational numbers.
  Example: $\left(\frac{1}{5} + \frac{2}{7}\right) + \frac{1}{3} = \frac{86}{105}$ and $\frac{1}{5} + \left(\frac{2}{7} + \frac{1}{3}\right) = \frac{86}{105}$
  $\left(\frac{3}{8} \times \frac{1}{9}\right) \times \frac{5}{7} = \frac{15}{504}$ and $\frac{3}{8} \times \left(\frac{1}{9} \times \frac{5}{7}\right) = \frac{15}{504}$
  $\left(\frac{4}{9} - \frac{3}{2}\right) - \frac{1}{3} = \frac{93}{57}$ but $\frac{4}{9} - \left(\frac{3}{2} - \frac{1}{3}\right) = \frac{39}{54}$
  $\left(\frac{3}{5} \div \frac{2}{5}\right) \div \frac{2}{5} = \frac{15}{4}$ but $\frac{3}{5} \div \left(\frac{2}{5} \div \frac{2}{5}\right) = \frac{3}{5}$

Addition of Rational Numbers

- **Case 1:** Adding rational numbers with same denominators:
  Example : $\frac{19}{5} + \frac{-7}{5}$
  $= \left(\frac{19-7}{5}\right) = \frac{12}{5}$
- **Case 2:** Adding rational numbers with different denominators:
  Example : $\frac{-3}{7} + \frac{2}{3}$
  LCM of 7 and 3 is 21
  So, $\frac{-3}{7} = \frac{-9}{21}$ and $\frac{2}{3} = \frac{14}{21}$
  $\Rightarrow \frac{-9}{21} + \frac{14}{21} = \left(\frac{-9+14}{21}\right) = \frac{5}{21}$

Subtraction of Rational Numbers

- To subtract two rational numbers, add the additive inverse of the rational number that is being subtracted, to the other rational number.
- Example: Subtract $\frac{2}{5}$ from $\frac{7}{9}$.

  $\frac{7}{9}$ + Additive Inverse of $\left(\frac{2}{5}\right)$

  $= \frac{7}{9} + \left(\frac{-2}{5}\right)$

  $= \left(\frac{35-18}{45}\right)$ {∵ LCM of 9 and 5 is 45}

  $= \frac{17}{45}$

Multiplication and Division of Rational Numbers

Multiplication of Rational Numbers

- **Case 1:** To multiply a rational number by a positive integer, multiply the numerator by that integer, keeping the denominator unchanged.

  $\frac{-3}{5} \times (7) = \frac{-3\times7}{5} = \frac{-21}{5}$

- **Case 2:** Steps to multiply one rational number by the other rational number:

  Step 1: Multiply the numerators of the two rational numbers.

  Step 2: Multiply the denominators of the two rational numbers.

  Step 3: Write the product as

  $\frac{Product\ of\ Numerators}{Product\ of\ Denominators}$

  $= \left(\frac{-5}{7}\right) \times \left(\frac{-9}{8}\right) = \frac{-5\times(-9)}{7\times8} = \frac{45}{56}$

Division of rational numbers

- To divide one rational number by the other rational numbers we multiply the rational number by the reciprocal of the other.

  Example: $\frac{-2}{3} \div \frac{1}{7}$

  $= \frac{-2}{3} \times$ Reciprocal of $\frac{1}{7}$

  $= \frac{-2}{3} \times 7$ {∵ Reciprocal of $\frac{1}{7} = 7$}

  $= \frac{-14}{3}$

Negatives and Reciprocals

- Rational numbers are classified as positive and negative rational numbers.
  (i) When both the numerator and denominator of a rational number are **positive integers or negative integers**, then it is a positive rational number.

  Example: $\frac{3}{5}$ is a positive rational number. $\frac{-3}{-5} = \frac{3}{5}$ is also a positive rational number.

  (ii) When either numerator or denominator of a rational number is a **negative integer**, it is a negative rational number.

  Example: $\frac{-3}{5} = -\frac{3}{5}$ is a negative rational number. $\frac{3}{-5} = -\frac{3}{5}$ is lso a negative rational number.

- If the product of two rational numbers is 1 then they are called **reciprocals** of each other.
  Example : $\frac{2}{3}$ is reciprocal of $\frac{3}{2}$, since $\frac{2}{3} \times \frac{3}{2} = 1$

  Note : The product of a rational number with its reciprocal is always 1.

Additive Inverse of a Rational Number

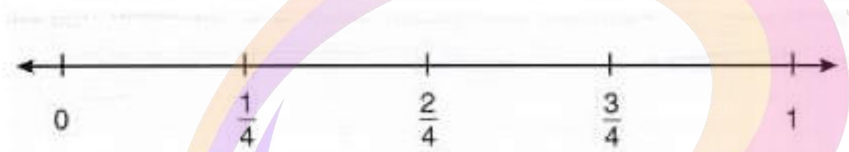
- Additive Inverse of a rational number $\frac{p}{q}$ is the number that, when added to $\frac{p}{q}$, yields zero.
  Example: Additive Inverse of a rational number $\frac{3}{5}$ is $\frac{-3}{5}$ and addtive inverse of $\frac{-3}{5}$ is $\frac{3}{5}$.
  Since $\frac{3}{5} + \frac{-3}{5} = 0$

Rational Numbers on a Number Line

- In order to represent a given rational number $\frac{a}{n}$, where a and n are integers, on the number line :

  Step 1 : Divide the distance between two consecutive integers into n parts.
  For example : If we are given a rational number $\frac{3}{4}$, we divide th space between 0 and 1, 1 and 2 etc. into **four** parts
  Step 2: Label the rational numbers till the range includes the number you need to mark

- The following figure shows how fractions $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ are represented on a number line.
- Divide the portion from 0 to 1 on the number line into four parts.
  Then each part represents $\frac{1}{4}^{th}$ portion of the whole.



Comparison of Rational Numbers

- **Case 1:** To compare two negative rational numbers, ignore their negative signs and reverse the order.
  Example: Which is greater: $\frac{-3}{8}$ or $\frac{-2}{7}$?
  Compare $\frac{3}{8}$ and $\frac{2}{7}$: $\frac{3}{8} > \frac{2}{7}$
  $\therefore \frac{-3}{8} < \frac{-2}{7}$
- **Case 2:** To compare a negative and a positive rational number, we consider that a negative rational number is to the left of zero whereas a positive rational number is to the right of zero on a number line. So, a negative rational number will always be less than a positive rational number.
  Example: (i) $\frac{-3}{11} < \frac{2}{5}$
  (ii) $\frac{-3}{8} < \frac{-2}{7}$